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MODELING OF THE FLOW PARTS OF STATIONARY CENTRIFUGAL COMPRESSOR MACHINES FOR COMPRESSION OF REAL GASES. GASDYNAMIC TESTS AND PROCESSING OF THEIR RESULTS. I. THEORETICAL CALCULATIONS

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Similarity numbers of the processes of compression of real gases in the stages of centrifugal compressor machines are given. A method for setting up promptly the thermal and calorific equations of state of the mixtures of real gases for the prescribed region of operation of a compressor machine is proposed. A procedure for processing of experimental data obtained in gasdynamic tests with mixtures of real gases and for reduction of the results obtained to the nominal conditions of operation of the flow part is presented.

1. In 1939, V. F. Ris [1] considered the problem of similarity numbers of the processes of compression of perfect gases in the case of adiabatic flows in the flow parts of centrifugal compressor machines. The engineering procedure of modeling of centrifugal compressor machines which was developed by him has made it possible to design and manufacture more than 300 different types of centrifugal compressor machines at the Neva Engineering Works alone [2].

To find the similarity numbers of flows in geometrically similar flow parts with the above-indicated assumptions of the properties of gases and in the absence of external heat exchange use was made of the π theorem of dimensional theory. Engineering practice has shown that, according to such similarity numbers as the Reynolds and Prandtl numbers and the ratio of heat capacities, perfect-gas flow can be considered to be self-similar in many cases and the basic, determining criteria are the flow coefficient and the conventional Mach number M_u if $M_u > 0.6$. The method of modeling developed is based on the fulfillment of the requirement of equality of the conventional Mach numbers and the flow coefficients in the model (initial) and designed (full-scale) geometrically similar flow parts. The employment of the method of modeling in creating new types of centrifugal compressor machines spares one an expensive operational development of flow parts with the aim of ensuring warranty parameters. A drawback of the method is its conservatism: a new machine cannot have a higher level of efficiency than its model. Improvement is possible only due to the calculation-theoretical investigations and gasdynamic tests of small-size models, but these tests are less difficult than tests and development work on full-scale machines. The bank of data obtained in testing models and in check tests of full-scale machines ensures designing centrifugal compressor machines with any parameters required by the customer of the machine; however, in recent decades one has more frequently met with cases where compressible gases and gas mixtures should be considered as real gases since the operating region must be located either near the right-hand boundary curve on the thermal diagram or at such high pressures where the equations of state significantly differ from the gas equations: the compressibility factor is $Z \neq 1.0$, while the isobaric heat capacity c_p depends on the temperature T and the pressure p. Such operating conditions are characteristic of the turbocompressors of steam refrigerating machines and centrifugal compressor machines for compression of natural gases conveyed by gas mains or pumped into underground storages. This determines the necessity of developing modeling methods for the processes of compression of real gases.

2. A system of the numbers of gasdynamic similarity of flows of viscous real gases can be obtained from consideration of the differential equations of continuity, motion, and energy balance which are written in dimensionless form without imposing in advance the restrictions on the properties of the gas, i.e., without taking into account the gas equations [3]:

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$$\mathrm{Sh}\,\frac{d\bar{\rho}}{d\bar{t}} + \bar{\nabla}\,(\bar{\rho}\,\bar{\mathbf{V}}) = 0\,, \tag{1}$$

$$\mathrm{Sh}\,\bar{\rho}\,\frac{d\bar{\mathbf{V}}}{d\bar{t}} + (\bar{\rho}\,\bar{\mathbf{V}}\,\bar{\nabla})\,\bar{\mathbf{V}} = \frac{1}{\mathrm{Fr}}\,\bar{\rho}\,\bar{\mathbf{F}} - \mathrm{Eu}\,\bar{\nabla}\,\bar{p} + \frac{1}{\mathrm{Re}}\,\bar{\nabla}\,\bar{\mathbf{T}}\,, \tag{2}$$

$$\mathrm{Sh}\,\bar{\rho}\,\frac{d\bar{i}}{d\bar{t}} + (\bar{\rho}\,\bar{\mathbf{V}}\,\bar{\nabla})\,\bar{i} = \frac{V_0^2}{i_0}\left\{\mathrm{Eu}\left[\frac{\partial\bar{p}}{\partial\bar{t}} + (\bar{\rho}\,\bar{\mathbf{V}}\,\bar{\nabla})\,\bar{p}\right] + \frac{1}{\mathrm{Re}}\,\bar{\mathbf{S}}\,\bar{\mathbf{T}} + \frac{1}{\mathrm{Pr}}\,\frac{c_{p0}T_0}{V_0^2}\,\bar{\nabla}\,(\bar{\lambda}\,\bar{\nabla}\,\bar{T})\right\}, \tag{3}$$

where the dimensionless tensor of viscous stresses $\bar{\mathbf{T}}$ is related to the dimensionless tensor of strain rates $\bar{\mathbf{S}}$ by the relation

$$\bar{\mathbf{T}} = 2\bar{\mu}\left(\bar{\mathbf{S}} + \frac{2}{3}\,\mathbf{E}\,\bar{\nabla}\,\bar{\mathbf{V}}\right);$$

here $\mathbf{E}$ is the tensor unit.

In the above equations, the dimensional quantities t, $\mathbf{V}$, ρ, p, T, ∇, i, μ, and λ are referred to the corresponding scales t_0, V_0, ρ_0, p_0, T_0, $1/L_0$, i_0, μ_0, and λ_0. In the general case, the equations describing real-gas flow contain seven unknowns, i.e., $\bar{\rho}$, $\bar{p}$, $\bar{T}$, $\bar{\mathbf{V}}$, $\bar{i}$, $\bar{\mu}$, and $\bar{\lambda}$, dependent on the spatial coordinates, time, similarity numbers (Strouhal, Froude, Euler, Reynolds, and Prandtl numbers), and dimensionless numbers V_0^2/i_0 and $c_{p0}T_0/i_0$. To close this system of equations one must supplement it with the thermal and calorific equations of state $f(p, \rho, T) = 0$ and $i = i(\rho, T)$ and the relations characterizing dynamic viscosity and thermal conductivity, i.e., $\mu = \mu(p, T)$ and $\lambda = \lambda(p, T)$.

Numerous forms of the thermal gas equation containing different numbers of constant coefficients have been proposed [4].

The similarity numbers involved in Eqs. (1)–(3) are related to the scales (which are selected when the quantities contained in the differential equations are made dimensionless) by the known relations

$$\mathrm{Sh} = \frac{L_0}{t_0V_0}\,,\quad \mathrm{Fr} = \frac{V_0^2}{L_0F_0}\,,\quad \mathrm{Eu} = \frac{p_0}{\rho_0V_0^2}\,,\quad \mathrm{Re} = \frac{\rho_0V_0L_0}{\mu_0}\,,\quad \mathrm{Pr} = \frac{\mu_0c_{p0}}{\lambda_0}\,.$$

The specific selection of the scale L_0, t_0, and others is governed by the distinctive features of the problem in question. In analyzing the operation of a turbocompressor stage [1], it is convenient to select the diameter of the impeller D_2 and the time of one revolution as the scales and, for example, the radial component of the average velocity of the flow in the exit cross section of the impeller $c_{\mathrm{r}2}$ as the velocity scale V_0. Then

$$\mathrm{Sh} = \frac{D_2\omega}{2\pi c_{\mathrm{r}2}} = \frac{u_2}{\pi c_{\mathrm{r}2}} = \frac{1}{\pi\varphi_{\mathrm{r}2}}\,,$$

where $\varphi_{\mathrm{r}2}$ is the flow coefficient of the impeller [1]. If the average velocity of the flow in the entrance cross section of the impeller c_0 is taken as the characteristic velocity, the flow coefficient $\varphi_{\mathrm{r}2}$ should be replaced by the flow coefficient $\varphi_0 = c_0/u_2$.

Having taken the values of the remaining quantities before the flow part of the stage p_{init}, T_{init}, ρ_{init}, μ_{init}, λ_{init}, and $c_{p\mathrm{init}}$ as their scales, we obtain

$$\mathrm{Eu} = \frac{p_{\mathrm{init}}}{\rho_{\mathrm{init}}u_2^2\varphi_{\mathrm{r}2}^2} = \frac{\mathrm{Eu}_u}{\varphi_{\mathrm{r}2}^2}\,,\quad \mathrm{Re} = \frac{\rho_{\mathrm{init}}D_2u_2}{\mu_{\mathrm{init}}}\,\varphi_{\mathrm{r}2} = \mathrm{Re}_u\varphi_{\mathrm{r}2}\,,\quad \mathrm{Pr} = \frac{\mu_{\mathrm{init}}c_{p\mathrm{init}}}{\lambda_{\mathrm{init}}}\,.$$

Here

$$\mathrm{Eu}_u = \frac{p_{\mathrm{init}}}{\rho_{\mathrm{init}} u_2^2}\,, \quad \mathrm{Re}_u = \frac{\rho_{\mathrm{init}} D_2 u_2}{\mu_{\mathrm{init}}}\,. \tag{4}$$

If the scale of the distribution density of body forces is $F_0 = \omega^2 D_2/2$, the Froude number is

$$\mathrm{Fr} = \frac{c_{\mathrm{r}2}^2 2}{D_2^2 \omega^2} = \frac{\varphi_{\mathrm{r}2}^2}{2}\,.$$

Having selected $i_0 = c_{p\mathrm{init}} T_{\mathrm{init}}$ as the enthalpy scale, we obtain

$$\frac{V_0^2}{i_0} = \frac{u_2^2}{c_{p\mathrm{init}} T_{\mathrm{init}}}\,\varphi_{\mathrm{r}2}^2\,.$$

The system of similarity numbers which follows from the differential equations (1)–(3) does not yet involve the Mach number M_u. This similarity number appears only after the introduction of the gas equations into consideration.

The conventional Mach number is

$$\mathrm{M}_u = u_2 / a_{\mathrm{init}}\,,$$

the isentropic velocity of sound before the flow part for real-gas flow is

$$a_{\mathrm{init}} = \sqrt{\left(\frac{dp}{d\rho}\right)_{s,\mathrm{init}}}\,;$$

on the other hand, we can assume that

$$a_{\mathrm{init}} = \sqrt{k_{\mathrm{init}} \frac{p_{\mathrm{init}}}{\rho_{\mathrm{init}}}}\,. \tag{5}$$

These two formulas lead us to the determination of the coefficient k_{init} in the real gas which is an isentropic exponent in the particular case of compression of a perfect gas and is equal to the ratio of the specific heat at constant pressure c_p and the specific heat at constant volume c_v. The problem of determination of k_{init} has been considered in [5] in greater detail. Having determined the value of the coefficient k_{init} in the real gas for the initial values of the parameters p and ρ, we can pass from the Eu_u number to the M_u number.

According to (4) and (5),

$$\mathrm{Eu}_u = 1/(k_{\mathrm{init}} \mathrm{M}_u^2)\,,$$

since

$$\mathrm{M}_u = u_2 \bigg/ \sqrt{k_{\mathrm{init}} \frac{p_{\mathrm{init}}}{\rho_{\mathrm{init}}}}\,.$$

The flow velocity is close to the velocity of sound in different elements of the flow part, i.e., local Mach numbers [1] depend on the specific geometry of the stage and the M_u number. When $\mathrm{M}_u < 0.5$ flow can be set self-similar in M_u, but when $\mathrm{M}_u > 0.5$ an increase in M_u leads to a change in the maximum efficiencies of the stage and deformation of the gasdynamic characteristics of the stage and the multistage section [1]. When $\mathrm{M}_u < 1.0$, k_{init} has a slight effect on the maximum value of the efficiency. In stationary centrifugal compressor machines, when $D_2 > 0.25$, the flow can usually be set self-similar in Re_u, too. The approximate procedure of taking into account the influence of

Re_u on the efficiency is contained on [6]. The problem of the influence of the Pr number on gasdynamic characteristics remains to be studied, in essence; however, taking into account the narrow range of variation of this number in gases, we can assume that Pr has no effect on the gasdynamic characteristics of centrifugal compressor machines.

In the case of three-dimensional flows of a viscous gas for high Reynolds numbers, even if the gas can be set perfect, it is still so difficult to solve the system of equations (1)–(3), supplemented with the relations closing it, in practice that the energy loss in the stage cannot be obtained by pure calculation. Therefore, in evaluating the efficiency and in engineering design calculations of centrifugal compressor machines, one has to rely on simplified procedures, experimental data, and modeling based on the observance of the geometric similarity of flow parts and on the equality of determining similarity numbers.

3. The engineering calculations of the flow parts of centrifugal compressor machines are carried out on the basis of one-dimensional equations of mass, energy, and the first law of thermodynamics which hold for an elementary streamline but extended to the entire cross section of the channel in question:

$$d\,(\rho c \sigma) = 0\,, \tag{6}$$

$$dH = dp/\rho + cdc + Tds\,, \tag{7}$$

$$di = dp/\rho + Tds - dq\,. \tag{8}$$

The last two relations yield that

$$dH = di + cdc + dq\,. \tag{9}$$

For the flow-part portion of finite length confined, for example, between the initial and final cross sections of the channel init–init and f–f, we have

$$H_{\text{init–f}} = i_{\text{f}} - i_{\text{init}} + 0.5\,(c_{\text{f}}^2 - c_{\text{init}}^2) + q_{\text{init–f}} \tag{10}$$

or

$$H_{\text{init–f}} = \int_{\text{init}}^{\text{f}} \frac{dp}{\rho} + 0.5\,(c_{\text{f}}^2 - c_{\text{init}}^2) + \int_{\text{init}}^{\text{f}} Tds\,. \tag{11}$$

In final form, instead of (6) we have

$$\rho c \sigma = G\,. \tag{12}$$

The relations given above contain the quantities ρ, c, p, T, i, H, s, and q to be determined. If it is taken that heat exchange between the flow and the surfaces confining it is absent or negligibly small, then $q_{\text{init–f}} = 0$ and for the closed system of equations to be obtained we must add to (6)–(9) or (10)–(12) four more dependences that determine the thermal and calorific properties of the gas and can be written in the form

$$p = p\,(\rho, T)\,, \tag{13}$$

$$i = \int_0^T c_p^0 dT - T^2 \int_0^\rho \left[\frac{\partial}{\partial T}\left(\frac{p}{T}\right)\right]_\rho \frac{d\rho}{\rho^2} + \frac{p}{\rho} - RT\,, \tag{14}$$

$$s = \int_0^T c_p^0 \frac{dT}{T} - R \ln T - \int_0^\rho \left(\frac{\partial p}{\partial T}\right)_\rho \frac{d\rho}{\rho^2} + \text{const}\,, \tag{15}$$

where c_p^0 is the specific heat at constant pressure in an ideally gas state. To obtain the last, fourth, relation we must make an assumption on the character of the process of compression, having taken, for example, that this process occurs in accordance with the power-law dependence

$$\frac{p}{\rho^m} = \frac{p_{\text{init}}}{\rho_{\text{init}}^m}, \tag{16}$$

or in accordance with [7] we must assume that the specific heat of the process is constant in a given regime of operation of the flow part. The latter case has been considered, for example, in [8]. Next we assume the validity of relation (16).

The specific work $H_{\text{init–f}}$ expended on compressing the gas is determined by formula (10). The increase in the potential and kinetic energies of the flow in traversal of the flow part is determined by the first two terms on the right-hand side of formula (11). The internal efficiency of the flow part η_{in} can be found using the formula

$$\eta_{\text{in}} = \frac{\int_{\text{init}}^{\text{f}} dp/\rho + 0.5\,(c_{\text{f}}^2 - c_{\text{init}}^2)}{i_{\text{f}} - i_{\text{init}} + 0.5\,(c_{\text{f}}^2 - c_{\text{init}}^2) + q_{\text{init–f}}} \tag{17}$$

or

$$\eta_{\text{in}} = 1 - \int_{\text{init}}^{\text{f}} Tds \Big/ \left[i_{\text{f}} - i_{\text{init}} + 0.5\,(c_{\text{f}}^2 - c_{\text{init}}^2) + q_{\text{init–f}} \right].$$

If $q_{\text{init–f}} = 0$ and the change in the kinetic energy $0.5(c_{\text{f}}^2 - c_{\text{init}}^2)$ is negligibly small as compared to the change in the enthalpy $i_{\text{f}} - i_{\text{init}}$ and the potential energy, then formula (17) yields the relation determining the polytropic efficiency:

$$\eta_{\text{pol}} = \int_{\text{init}}^{\text{f}} \frac{dp}{\rho} \Big/ (i_{\text{f}} - i_{\text{init}}) . \tag{18}$$

As a definition of the polytropic efficiency, a differential expression following from [8] is recommended in [6]:

$$\eta_{\text{pol}} = (dp/\rho)/di .$$

This expression also yields formula (18).

When the power-law approximation (described by relation (16)) of the process of compression is employed we have

$$\eta_{\text{pol}} = \frac{m}{m-1} \frac{p_{\text{init}}}{\rho_{\text{init}}} \left[\left(\frac{p_{\text{f}}}{p_{\text{init}}} \right)^{\frac{m-1}{m}} - 1 \right] \Big/ (i_{\text{f}} - i_{\text{init}}) . \tag{19}$$

Formula (19) is substantially simplified in the case of compression of a perfect gas. The assumption of constancy of η_{pol} along the flow part also simplifies the calculations.

4. In testing centrifugal compressor machines, one measures the values of p_{init} and p_{f} and T_{init} and T_{f} for different mass flow rates G and rotational frequencies of the rotor n to obtain gasdynamic characteristics, i.e., the dependences of η_{pol}, the ratio of the pressures $\pi_{\text{f}} = p_{\text{f}}/p_{\text{init}}$, and the internal power $N_{\text{in}} = G(i_{\text{f}} - i_{\text{init}})$ on the volumetric capacity $Q = G/\rho_{\text{init}}$. The gas analysis must also establish the composition of the gas mixture in each regime of operation of the machine. The latter issue virtually does not arise in testing air machines; however, the experience accumulated in testing centrifugal compressor machines for compression of different gas mixtures at petrochemical

enterprises points to the fact that the composition of the gas can substantially change in the course of the tests. Therefore, to process experimental data and to reduce them to the nominal operating conditions of the machine one must have a procedure for setting up promptly the equations of state of a gas mixture for each operating regime in testing. This problem must be solved in the course of the tests themselves so as not to retard the processing of the data obtained. One possible way of setting up the thermal equation of state of a gas mixture follows from [9], where it is shown how the Benedict–Webb–Rubin (BWR) equation can be simplified by discarding the exponential terms. Then a simplified BWR equation describes rather well the thermal properties of a real gas in the required region of operation of the compressor (this region is always known).

The simplified BWR equation can be written in the form

$$Z = 1 + (\bar{B} + \bar{A}/T_{\rm red} + \bar{C}/T_{\rm red}^3)\, p_{\rm red}/(ZT_{\rm red}) + (\bar{b} + \bar{a}/T_{\rm red} + \bar{c}/T_{\rm red}^3)\, [p_{\rm red}/(ZT_{\rm red})]^2 . \tag{20}$$

Here

$$Z = p/(\rho RT), \quad T_{\rm red} = T/T_{\rm cr}, \quad p_{\rm red} = p/p_{\rm cr},$$

in this case $p_{\rm cr}$ and $T_{\rm cr}$ are the pseudocritical parameters of the gas mixture.

The coefficients $\bar{A}$, $\bar{B}$, ..., $\bar{b}$, and $\bar{c}$ can be computed from the values of the compressibility coefficient which have been found by the Lie–Kessler method [3, 4]; the tabular values of Z are contained in [4, 6]. They are given for different values of the reduced parameters $p_{\rm red}$ and $T_{\rm red}$. The pseudocritical parameters of the mixture are computed from the volume fractions of its components r_j, the known critical parameters $p_{{\rm cr}j}$ and $T_{{\rm cr}j}$ of these components, and the acentricity coefficient ω_j. The values of $p_{{\rm cr}j}$, $T_{{\rm cr}j}$, and ω_j for numerous substances are indicated, for example, in [4]. When the data on the acentricity coefficient ω_j are absent its value can be found from the normal boiling temperature of a substance and the values of $p_{{\rm cr}j}$ and $T_{{\rm cr}j}$ [3, 4]. To determine the coefficients of Eq. (20) that correspond to a given composition of the gas mixture one must find its pseudocritical parameters and the acentricity coefficient, select arbitrarily six pairs of the values of $p_{\rm red}$ and $T_{\rm red}$ located in the operating region of the compressor, and set up a system of algebraic equations which makes it possible to find the coefficients of Eq. (20). Employing the Lie–Kessler tables available in [4, 6], from the pairs of the values of $p_{\rm red}$ and $T_{\rm red}$ one should find Z computed from the formula

$$Z = Z^{(0)} + \omega Z^{(1)} .$$

Here $Z^{(0)}$ and $Z^{(1)}$ are the quantities determined from the tables for the selected values of $p_{\rm red}$ and $T_{\rm red}$. The system of linear equations for $\bar{A}$, $\bar{B}$, ..., $\bar{b}$, and $\bar{c}$ has the form

$$\bar{B} + \frac{1}{T_{{\rm red}i}}\bar{A} + \frac{1}{T_{{\rm red}i}^3}\bar{C} + \left(\bar{b} + \frac{1}{T_{{\rm red}i}}\bar{a} + \frac{1}{T_{{\rm red}i}^3}\bar{c}\right)\frac{p_{{\rm red}i}}{Z_i T_{{\rm red}i}} = \frac{Z_i T_{{\rm red}i}}{p_{{\rm red}i}}(Z_i - 1), \quad i = 1, 2, ..., 6$$

and it is solved using a standard program.

In dimensional form, the thermal equation of state is as follows:

$$p = RT\rho\left[1 + \left(\bar{B} + \frac{\bar{A}T_{\rm cr}}{T} + \frac{\bar{C}T_{\rm cr}^3}{T^3}\right)\frac{RT_{\rm cr}}{p_{\rm cr}}\rho + \left(\bar{b} + \frac{\bar{a}T_{\rm cr}}{T} + \frac{\bar{c}T_{\rm cr}^3}{T^3}\right)\left(\frac{RT_{\rm cr}}{p_{\rm cr}}\right)^2\rho^2\right]. \tag{21}$$

In accordance with (14) we have

$$i = \left(a_0 + \frac{a_1 T}{2} + \frac{a_2 T^2}{3} + \frac{a_3 T^3}{4}\right)T$$

$$+ \left[\left(\bar{A} + \frac{3\bar{C}T_{\rm cr}^2}{T^2}\right)\frac{RT_{\rm cr}}{p_{\rm cr}}\rho + \left(\bar{a} + \frac{3\bar{c}T_{\rm cr}^2}{T^2}\right)\left(\frac{RT_{\rm cr}}{p_{\rm cr}}\right)^2\frac{\rho^2}{2} + (Z - 1)\frac{T}{T_{\rm cr}}\right]RT_{\rm cr}, \tag{22}$$

here a_0, a_1, a_2, and a_3 are the coefficients of the approximation polynomial for the specific heat at constant pressure of the gas mixture in an ideal gas state $c_p^0 = a_0 + a_1 T + a_2 T^2 + a_3 T^3$. These coefficients are calculated from the volume fractions of the components r_j, their molecular masses, and the corresponding coefficients for the specific heat at constant pressure a_{0j}, a_{1j}, a_{2j}, and a_{3j} of these components. The values of the coefficients a_0, a_1, a_2, and a_3 for a large number of substances are given in [4].

When Eq. (20) is employed we have

$$k = \frac{1}{Z}\left\{1 + 2\left(\bar{B} + \bar{A}\frac{T_{\text{cr}}}{T} + \bar{C}\frac{T_{\text{cr}}^3}{T^3}\right)\frac{RT_{\text{cr}}}{p_{\text{cr}}}\rho + 3\left(\bar{b} + \bar{a}\frac{T_{\text{cr}}}{T} + \bar{c}\frac{T_{\text{cr}}^3}{T^3}\right)\left(\frac{RT_{\text{cr}}}{p_{\text{cr}}}\right)^2\rho^2\right.$$

$$\left.+\frac{\left[1 + \left(\bar{B} - 2\bar{C}\frac{T_{\text{cr}}^3}{T^3}\right)\frac{RT_{\text{cr}}}{p_{\text{cr}}}\rho + \left(\bar{b} - 2\bar{c}\frac{T_{\text{cr}}^3}{T^3}\right)\left(\frac{RT_{\text{cr}}}{p_{\text{cr}}}\right)^2\rho^2\right]^2}{\frac{c_p^0}{R} - 1 + 3\left(2\bar{C} + \bar{c}\frac{RT_{\text{cr}}}{p_{\text{cr}}}\rho\right)\frac{RT_{\text{cr}}}{p_{\text{cr}}}\rho}\right\}.$$

To determine the volumetric capacity Q one must find in advance the initial density ρ_{init} from the experimental values of p_{init} and T_{init}. For this purpose Eq. (21) is employed. The same equation makes it possible to compute, from the experimental values of p_{f} and T_{f}, the final density ρ_{f} and then the exponent m in relation (16):

$$m = \ln\left(\frac{p_{\text{f}}}{p_{\text{init}}}\right) \Big/ \ln\left(\frac{\rho_{\text{f}}}{\rho_{\text{init}}}\right). \tag{23}$$

The initial and final enthalpies i_{init} and i_{f} are computed using formula (22). Thereafter one can find η_{pol} using (19).

5. In accordance with the theorem of a change in the momentum of the gas in the impeller [1, 8] in the absence of heat exchange between the flow and the walls confining it and $0.5(c_{\text{f}}^2 - c_{\text{init}}^2) << i_{\text{f}} - i_{\text{init}}$, the specific work expended on compressing the gas in the stage is

$$H_{\text{init–f}} = i_{\text{f}} - i_{\text{init}} = u_2^2\chi . \tag{24}$$

Here χ is the coefficient of power dependent on the geometry and the flow coefficient φ_{r2} in the case of subsonic gas flow in the impeller. Therefore, according to formula (19), we have

$$\eta_{\text{pol}} = \frac{m}{m-1}\frac{\text{Eu}_u}{\chi}\left(\pi_{\text{f}}^{\frac{m-1}{m}} - 1\right). \tag{25}$$

Relation (16) yields that the coefficient of change of the specific volume of the gas in the flow part is

$$k_v = \frac{\rho_{\text{f}}}{\rho_{\text{init}}} = \pi_{\text{f}}^{1/m} . \tag{26}$$

Relations (25) and (26) provide a basis for engineering modeling of geometrically similar flow parts which are intended for operation with different gases for different initial conditions and rotational frequencies of the rotors. The mass capacity of the stage G is related to the impeller width b_2 and other characteristic quantities by the obvious relation [1]

$$G = \pi b_2 D_2 u_2 \varphi_{r2} k_{v2}\, \rho_{\text{init}} ,$$

in which k_{v2} is the coefficient of change of the specific volume of the gas in the impeller on the portion between the initial cross section of the flow part and the exit cross section of the impeller. The internal power of the stage is

$$N_{\text{in}} = \pi \frac{b_2}{D_2} \varphi_{r2}\, \chi k_{v2}\, \rho_{\text{init}} D_2^2 u_2^3 .$$

Consequently, under identical operating regimes of two geometrically similar stages, i.e., for the same flow coefficients φ_{r2}, the same conventional Euler numbers Eu_u, and the same m, the polytropic efficiencies η_{pol}, the pressure ratios π_{f}, and the values of $N_{\text{in}}/(\rho_{\text{init}} D_2^2 u_2^3)$ must turn out to be the same. This holds in the presence of the self-similarity of flows in conventional Re_u numbers and other numbers of gasdynamic similarity. If two flow parts are geometrically similar but differ in the impeller diameters D_2 and D_2', i.e., one is made on the scale $I = D_2/D_2'$ in relation to the other, then under the identical operating regimes $\eta_{\text{pol}}' = \eta_{\text{pol}}$, the volumetric capacities Q', the increases in the enthalpy $i_{\text{f}}' - i_{\text{init}}'$ and $i_{\text{f}} - i_{\text{init}}$, and the pressure ratios π_{f}' and π_{f} turn out to be related by the relations

$$Q' = I^3 \frac{n'}{n} Q , \quad i_{\text{f}}' - i_{\text{init}}' = I^2 \left(\frac{n'}{n}\right)^2 (i_{\text{f}} - i_{\text{init}}) , \quad \pi_{\text{f}}' = \pi_{\text{f}} , \quad N_{\text{in}}' = I^5 \frac{\rho_{\text{init}}'}{\rho_{\text{init}}} \left(\frac{n'}{n}\right)^3 N_{\text{in}} .$$

The fulfillment of the condition $m = m'$ is required for the total similarity of the processes of compression; however, in compression of real gases, this requirement cannot be fulfilled in practice even in the case of compression of one and the same gas but with different $p_{\text{init}}/\rho_{\text{init}}$ and $p_{\text{init}}'/\rho_{\text{init}}'$. Therefore, the rigorous fulfillment of the condition $\text{Eu}_u' = \text{Eu}_u$ has to be abandoned in favor of the fulfillment of the condition $k_v' = k_v$. Assuming, as previously, that $\eta_{\text{pol}'} = \eta_{\text{pol}}$, we obtain

$$\text{Eu}_u' = \beta_m^2\, \text{Eu}_u , \tag{27}$$

where

$$\beta_m = \sqrt{\frac{m}{m-1}\left[(\pi_{\text{f}}')^{\frac{m}{m'}\cdot\frac{m-1}{m}} - 1\right] \Big/ \left\{\frac{m'}{m'-1}\left[(\pi_{\text{f}}')^{\frac{m'-1}{m'}} - 1\right]\right\}} .$$

The employment of this condition is equivalent to the assumption of a weak influence of Eu_u (with a small change in it) on η_{pol} for $\varphi_{r2}' = \varphi_{r2}$. For higher values of Eu_u corresponding to $\text{M}_u < 0.7$ this assumption can be considered to be quite justified.

Taking into account that $p_{\text{init}}/\rho_{\text{init}} = Z_{\text{init}} R T_{\text{init}}$, from condition (27) we obtain

$$\frac{n'}{n} = \frac{1}{I\beta_m} \sqrt{\frac{Z_{\text{init}}' R' T_{\text{init}}'}{Z_{\text{init}} R T_{\text{init}}}} \tag{28}$$

and the modeling scale

$$I = \beta_m \sqrt[4]{\frac{Z_{\text{init}} R T_{\text{init}}}{Z_{\text{init}}' R' T_{\text{init}}'}} \sqrt{\frac{Q'}{Q}} . \tag{29}$$

In designing a new machine by the modeling method, Q', π'_f, and the initial conditions for it are prescribed; only m' is unknown. The problem is reduced to selection of the design point on the available hydrodynamic characteristics of the initial model. It must be found at a sufficient distance from the boundary of stalling and surging and the efficiency at it must be sufficiently high. The selection of the design point on the hydrodynamic characteristics of the model makes it possible to find Q, π_f, n, χ, η_{pol}, and m. This enables us to calculate β_m and find m'. Next we determine the modeling scale I and the rotational frequency of the rotor of the designed machine n'. For the modeling method to be employed the designer must have at his disposal a set of hydrodynamic characteristics for several flow parts that can be employed as model ones.

The problem of determination of the "equivalent" rotational frequency n_e in bench tests of a new machine operating with air or other modeling gas is adjacent to the problem of selection of the modeling scale. In this case

$$I = 1\,, \quad n_e = n'\beta_m \sqrt{\frac{Z_{init} R T_{init}}{Z'_{init} R' T'_{init}}}\ .$$

The coefficient β_m is calculated for the design regime of operation of the flow part. In the case where a centrifugal compressor machine is intended for operation with a gas heavier than air the equivalent rotational frequency can turn out to be higher than the operating frequency n' and to ensure the strength of the rotor in bench tests it will be required to manufacture the rotor from stronger materials or to carry out tests on a closed test loop with a modeling gas heavier than air.

Formulas to convert the hydrodynamic characteristics to another rotational frequency n'' differing from the equivalent one are based on the violation of the requirement $k'_v = k''_v$ and the similarity of Eu'_u and Eu''_u. When the conversion formulas are used one actually assumes the self-similarity in the Eu_u number. For identical regimes in the case of one and the same machine we have

$$Q' = \frac{n''}{n'} Q' \frac{k''_{v2}}{k'_{v2}}\,, \quad i''_f - i''_{init} = \left(\frac{n''}{n'}\right)^2 \left(i'_f - i'_{init}\right), \quad \pi''_f = \left[1 + \frac{m'' - 1}{m''} \frac{\rho''_{init}}{p''_{init}} \eta'_{pol} \left(i''_f - i''_{init}\right)\right]^{\frac{m''}{m''-1}}. \tag{30}$$

It is assumed that $\eta''_{pol} = \eta'_{pol}$.

These formulas yield rather good results in conversions of the hydrodynamic characteristics of single-stage machines [8] but in the case of multistage uncooled sections they provide only approximate results. For a more accurate conversion one must determine the dimensionless hydrodynamic characteristics of all the stages from experimental data and from these characteristics calculate the total dimensional hydrodynamic characteristics of the section under new operating conditions.

The conversions for $m'' \neq m'$ involve difficulties associated with the determination of m''. As a result of the conversion the increase in the enthalpy $\Delta i'' = i''_f - i''_{init}$ turns out to be known but it is necessary to know the final density ρ''_f and final temperature T''_f and to solve the system of equations which enables us to determine m''. For this purpose we have to obtain the dependences $\rho''_f(T''_f)$, $p''_f(T''_f)$, and $m''(T''_f)$ and only after that to determine the sought value of π''_f.

With the known $\Delta i''$, the thermal and calorific equations of state (21) and (22) yield the quadratic equation for ρ''_f:

$$\bar{\rho}''^2_f + 2\,\frac{2\bar{A}\,T''^2_{red.f} + \bar{B}\,T''^3_{red.f} + 4\bar{C}}{3\bar{a}\,T''^2_{red.f} + 2\bar{b}\,T''^3_{red.f} + 5\bar{c}}\,\bar{\rho}''_f + \frac{2T''^2_{red.f}}{R''\,T''_{cr}}\,\frac{\Delta i'' + i''_{init} - i^{0''}_f}{3\bar{a}\,T''^2_{red.f} + 2\bar{b}\,T''^3_{red.f} + 5\bar{c}} = 0\,,$$

in which the enthalpy of the end of compression for an ideal gas state is

$$i_f^{0''} = \left(a_0'' + \frac{a_1''}{2} T_f'' + \frac{a_2''}{3} T_f''^2 + \frac{a_3''}{4} T_f''^3 \right) T_f'' , \quad \bar{\rho}_f'' = \frac{R'' T_{cr}''}{p_{cr}''} \rho_f'' ; \quad T_{red.f}'' = \frac{T_f''}{T_{cr}''} .$$

The final pressure is determined by Eq. (21), while the value of m is determined by formula (23). The graphical method of solution of the problem is obvious. Having constructed the dependences $m''(T_f'')$ and $\pi_f''(T_f'')$, we can find the point of intersection of $\pi_f''(T_f'')$ and the curve constructed in accordance with formula (30).

The method of determination of m and π_f can be programmed for calculation on a personal computer; then graphical constructions become unnecessary.

NOTATION

ρ, p, and T, density, pressure, and temperature; t, time; **V**, velocity; ∇, Hamilton operator; **F**, distribution density of body forces; **T** and **S**, tensors of viscous stresses and strain rates; μ, dynamic viscosity; λ, thermal conductivity; c_p, specific heat at constant pressure; i and s, enthalpy and entropy; a, isentropic velocity of sound; k, coefficient in the formula for the velocity of sound; L_0, linear scale in making the quantities dimensionless; Sh, Fr, Re, Eu, Pr, and M, Strouhal, Froude, Reynolds, Euler, Prandtl, and Mach numbers; D_2, diameter of the impeller; u_2, circular velocity; c_{r2}, radial component of the velocity at the exit from the impeller; φ_{r2}, flow coefficient; ω, rotational velocity; ω_j, acentricity coefficient; σ, cross-sectional area; c, average velocity of the gas; H, work expended on compressing; q, heat removed from the gas; η_{in} and η_{pol}, internal and polytropic efficiencies; N_{in}, internal power; χ, power factor; π_f, pressure ratio; m, exponent of the process of compression; Re_u, Eu_u, and M_u, conventional Reynolds, Euler, and Mach numbers; G and Q, mass and volumetric flow rates; I, modeling scale; n, rotational frequency; $\bar{A}$, $\bar{B}$, $\bar{C}$, $\bar{a}$, $\bar{b}$, and $\bar{c}$, coefficients of the simplified dimensionless BWR equation; R, gas constant; r_j, volume concentration of the jth component of the gas mixture; p_{cr} and T_{cr}, critical parameters of the gas or pseudocritical parameters of the gas mixture; p_{red} and T_{red}, parameters referred to p_{cr} and T_{cr} respectively. Subscripts and superscripts: 0, scales of the quantities in making them dimensionless; –, dimensional quantity referred to its scale; init and f, quantities on the initial and final cross sections of the flow part; r, radial; in, internal; pol; polytropic; red, reduced; cr, critical; e, equivalent.

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